



An optimal method for GNSS water vapor tomography based on parameter adaptation

Fei Yang^{1,2} · Ruixian Hao¹ · Haoyu Wang¹ · Lei Wang^{2,3} · Zhicai Li¹ · Junxi Zheng¹ · Yingying Wang¹ · Ran Chen¹

Received: 17 January 2024 / Accepted: 26 November 2025 / Published online: 10 December 2025
© The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2025

Abstract

Precipitable water vapor (PWV) reflects the vertically integrated amount of water vapor in the atmosphere, but it only represents the two-dimensional distribution of water vapor. With the development of GNSS satellite constellation and station network, GNSS water vapor tomography has emerged as a prominent technique for obtaining the three-dimensional water vapor distribution and become a hotspot in GNSS meteorology. We propose an optimal method for GNSS water vapor tomography based on parameter adaptation to solve the ill-posed problem caused by the discretization parameter redundancy in the tomographic model. This method introduces the minimum probability factor of voxels being penetrated by signals, determines the number and position of the redundant parameters in the observation equation by iterating the coefficient matrix, establishes the optimal tomographic model by eliminating the zero elements of the coefficient matrix and the redundant parameters, and calculates the water vapor density of all voxels by constructing a Laplace smoothing equation. The tomography experiments conducted in Hong Kong show that the average root mean square error (RMSE) and mean absolute error (MAE) for the proposed and traditional method are 1.25/0.70 and 1.83/0.92 mm in the internal accurate testing, respectively, compared to the GAMIT-estimated slant water vapor (SWV) as a reference. The average RMSE and normalized RMSE are 10.9 mm and 8.2% for the proposed method, while these values are 13.2 mm and 10% for the traditional method. Using the radiosonde data as references, the proposed method improves average RMSE and MAE by 20.4% and 25.3% compared with the traditional method.

Keywords GNSS meteorology · Atmospheric water vapor · Tomography · Radiosonde

Introduction

Water vapor is the predominant greenhouse gas in the Earth's atmosphere, crucial for maintaining a temperature environment conducive to supporting life and playing an important role in global warming (Yao et al. 2017a; Sun et al. 2023). Its distribution and dynamics exert a substantial influence on climate and weather change. Meanwhile, water vapor

has a delayed effect on the propagation of electromagnetic waves through the atmosphere, making it one of the main error sources for satellite-based Earth observations. Therefore, the precise monitoring of water vapor, characterized by high temporal and spatial resolutions, has consistently remained a central objective within the field of meteorology (Zhao et al. 2022; Huang et al. 2022).

With the development of the Global Navigation Satellite System (GNSS) and its continuous expansion in related fields, the spatiotemporal distribution of water vapor can be inverted through the use of a tomographic technique, utilizing densely distributed GNSS sites. This concept was first proposed by Braun et al. (1999) and then was realized by Flores et al. (2000). GNSS water vapor tomography has become a promising technique to achieve the water vapor information, offering several advantages such as all-weather, continuous operation, high spatiotemporal resolution (Yang et al. 2023a). In GNSS water vapor tomography, the study region covered by ground GNSS receivers should

✉ Fei Yang
yangfei@cumtb.edu.cn

¹ College of Geoscience and Surveying Engineering, China University of Mining and Technology-Beijing, Beijing 100083, China

² Hubei LuoJia Laboratory, Wuhan 430079, China

³ State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, Wuhan University, Wuhan 430079, China

be discretized into finite cubic closed voxels according to longitude, latitude and altitude. Within each voxel and specific time period the estimated parameter like wet refractivity is assumed to be constant. The observations for water vapor tomography are the slant water vapor (SWV) derived from the GNSS data, which is the precipitable water in the direction of the signal ray path.

Due to the limited distribution of GNSS sites and the fixed structure of the satellite constellation, the geometric distribution of the observed signals utilized in the GNSS water vapor tomography is an inverted cone, which creates a spatial geometric mismatch with the box shaped tomographic area. This results in a large number of voxels in the water vapor tomographic model that cannot be penetrated by satellite signals or are only passed through by a very small number of signals. Furthermore, a substantial proportion of elements in the coefficient matrix of the water vapor tomographic model become zero, potentially leading to the formation of a sparse matrix. This phenomenon is commonly referred to as the ill-posed problem in GNSS water vapor tomography, which constitutes the central focus of current research efforts. Various scholars have investigated the ill-posed problem in GNSS water vapor tomography, proposing several approaches that can be categorized into four main types:

(1) Expanding GNSS observations. Fan et al. (2021) explored the dynamic changes of the tomographic top layer and constructed a vertical non-uniform stratification method; Yao et al. (2017b) proposed the non-uniform symmetrical division of horizontal voxels; Trzcina et al. (2019) and Sa et al. (2021) adopted a voxel buffer around each side of the three-dimensional grid. The above methods enable more signals to pass through the top of the tomographic region, expanding the number of GNSS observations that can be used for tomographic modeling. However, they may have limitations in terms of specific tomographic regions and time periods, resulting in limited applicability. On the other hand, Adavi et al. (2014) introduced the virtual GNSS sites to increase the number of signals at specific locations Yang et al. (2018); Zhang et al. (2021a, b) utilized the ray tracing and HGOT to construct the virtual GNSS observations, respectively; Zhao et al. 2017; Zhang et al. 2020a) incorporated the GNSS signals penetrating from the side face into the tomographic model based on the unit factor of water vapor and height factor model, respectively; Zhao et al. (2021) extended the observations from the GNSS sites outside the tomographic region to the tomographic model using the truncation factor model and GPT2w model, respectively. The above methods increase the number of various GNSS observations, but they all rely on external data or models.

(2) Integrating other types of observations, such as data from COSMIC occultation (Ye et al. 2016), GNSS-R (Shafei et al., 2020), InSAR (Benevides et al. 2016; Heublein et al. 2019), numerical weather prediction (Trzcina et al., 2019), WRF model (Haji-Aghajany et al. 2020), meteorological reanalysis (Yang et al. 2023b), FengYun and MODIS (Zhang et al. 2021a, b). Multi-source observations as mentioned above could improve the ill-posed problem, but their spatiotemporal resolution, availability, and matching with the tomographic region also restrict the construction of the fusion model.

(3) Introducing optimal algorithms. The singular value decomposition, damped least squares and Tikhonov regularization was adopted based on the traditional least squares (Adavi et al., 2014; Benevides et al. 2016; Shafei et al., 2020), but the fluctuation of small singular values, the change of the damping and the selection of regularization parameters in these methods become the key factors affecting the water vapor tomography. Based on the assumption that the water vapor of the voxel satisfies a Gauss Markov random walk in a certain time, Gradinarsk and Jariemark (2004) first utilized the Kalman filter method in water vapor tomography, and Jiang et al. (2013) and Rohm et al. (2014) successively proposed the adaptive Kalman filter and the robust Kalman filter for the water vapor tomography, respectively. They need to initialize the filter and provide initial covariance for the state equation which depends on external data. On the other hand, Bender et al. (2011) proposed an iterative algorithm based on algebraic reconstruction to avoid the inversion of sparse matrix, He et al. (2015) discussed the performances of the ART, MART and SIRT in water vapor tomography. Then also improved iterative algorithms were developed, such as the parallel algebraic reconstruction and the adaptive simultaneous iterative reconstruction (Sa et al. 2021; Zhang et al. 2020b). These iterative algorithms often only iteratively update water vapor on some voxels, and the accuracy of the tomographic results largely depends on the accuracy of the initial values. In addition, Haji-Aghajany et al. (2020) utilized the function-based water vapor tomography based on B-spline function; The compressive sensing reconstruction (Heublein et al. 2019), the optimization algorithm (Yang et al. 2020a) and the artificial neural network algorithm (Razin and Voosoghi 2020) have also been tried to improve the ill-posed problem for the GNSS water vapor tomography.

The above three types of methods not only have their own inherent shortcomings, but also have the problem of discretization parameter redundancy in the water vapor tomographic model. This relates to the fact that voxels that are not passed by signals still participate in establishing the tomographic model. These redundant parameters do not have any connection with the GNSS observations, which

is the fundamental cause of the ill-posed problem in GNSS water vapor tomography. While existing methods attempt to mitigate the symptoms (e.g., by adding data or applying generic constraints), they do not directly address this root cause. Therefore, to fundamentally solve the ill-posed problem, this paper proposed an optimal method for GNSS water vapor tomography based on parameter adaptation. The conventional water vapor tomographic model is enhanced by introducing the minimum probability factor that indicates the likelihood of voxels being penetrated by signals. By iteratively analyzing the tomographic coefficient matrix, the number and location of redundant parameters in the tomographic observation equation are determined. Subsequently, an optimal tomographic model is constructed by adaptively eliminating zero elements from the coefficient matrix and removing redundant parameters. This approach directly targets and eliminates the source of the ill-posedness, setting it apart from previous strategies.

To estimate the water vapor content of all voxels, a Laplace smoothing equation that reflects the relationship between redundant parameters and solved parameters is constructed and calculated using the least squares method.

Methodology

Troposphere estimation

The observation in water vapor tomography is SWV, which can be converted from the slant wet delay (SWD) using the following formula (Bevis et al. 1992; Wang et al. 2025):

$$SWV = \Pi \times SWD = \frac{10^6}{\rho_w \times \frac{R}{m_w} \left(\frac{k_3}{T_m} + k_2 - \frac{m_w}{m_d} \times k_1 \right)} \times SWD, \quad (1)$$

where Π refers to a conversion factor, ρ_w denotes the liquid water density, m_w and m_d are the molar mass of the wet and dry atmosphere, respectively. k_1 , k_2 , k_3 are the empirical physical constants and R represents the universal gas constant. T_m indicates the weighted mean temperature, which can be calculated using the surface temperature (Bevis et al. 1992; Yang et al. 2022). SWD can be obtained by mapping the wet delay gradients and the zenith wet delay (ZWD) into the slant direction based on the formula expressed as follows:

$$SWD = f(ele) \times ZWD + f(ele) \times \cot(ele) \times (G_{NS}^w \times \cos(azi) + G_{WE}^w \times \sin(azi)), \quad (2)$$

where azi and ele denote the satellite azimuth and elevation, respectively. f indicates the wet mapping function, G_{NS}^w and G_{WE}^w are the wet delay gradient parameters in the north-south and east-west direction, respectively. Affected by water vapor along the signal ray, ZWD represents the zenith wet delay which is the wet component of the zenith total delay (ZTD) (Yang et al. 2021; Li et al. 2024). As the primary parameter retrieved from GNSS data processing, ZTD is utilized to achieve ZWD by subtracting the zenith hydrostatic delay (ZHD) from it (Li et al. 2025). The ZHD can be accurately calculated, e.g. by means of the Saastamoinen model as follows (Saastamoinen 1972; Huang et al. 2023):

$$ZHD = \frac{0.002277 \times P}{1 - 0.00266 \times \cos(2\varphi) - 0.00028 \times H}, \quad (3)$$

where P denotes the surface pressure, φ and H refer to the latitude and geodetic height of the site, respectively.

The proposed water vapor tomographic method

The initial observation equations of the water vapor tomography can be established using the SWV calculated by (1) and the intercepts of the signal rays in each voxel. Unlike the traditional method which directly solves the full tomographic observation equations containing numerous redundant parameters via least squares, the proposed method introduces a parameter adaptation process. Specifically, the minimum probability factor of voxels being penetrated by signals is employed to loop traverse the coefficient matrix of the tomographic observation equations, thereby determining the number and position of the redundant parameters and establishing the optimal tomographic observation equations. By using a Laplace smoothing equation that reflects the relationship between redundant parameters and solved parameters, the water vapor of all voxels could be achieved using the least squares method. The specific steps involved in this process can be expressed as follows:

(1) The research area is divided into voxels according to the number and distribution of the GNSS sites. The SWV corresponded to the GNSS signals passing through the top layer during the tomographic period is calculated as well as the intercept of the signals in the corresponding voxel.

(2) The initial observation equations are constructed using the SWV and the intercepts as follows:

$$\begin{bmatrix} a_1^1 & a_1^2 & \cdots & a_1^n \\ a_2^1 & a_2^2 & \cdots & a_2^n \\ \vdots & \vdots & \ddots & \vdots \\ a_l^1 & a_l^2 & \cdots & a_l^n \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} SWV_1 \\ SWV_2 \\ \vdots \\ SWV_l \end{bmatrix} \quad (4)$$

where a_n^l represents the intercept of the l -th satellite signal passing through the n -th voxel, forming the coefficient matrix of the tomographic model. x_n refers to the water vapor density of n -th voxel, which are the parameters to be solved.

(3) The minimum probability factor of voxels being penetrated by signals is calculated by the following formula:

$$\beta = s/n \quad (5)$$

where s and n denote the number of vertical voxels and the total number of voxels in the tomographic model, respectively. This factor serves as a data-driven threshold to identify voxels with insufficient ray coverage, ensuring that only parameters with meaningful observational support are retained in the model.

(4) The coefficient matrix is traversed and the proportion of the non-zero elements in each column is counted. The proportion of each column is then compared with the minimum probability factor. If the proportional value is large, the corresponding column vector is retained in the coefficient matrix; otherwise, the column vector is removed. Then the deleted column vectors are traversed to identify the positions with a value of 1. These positions correspond to rows in the coefficient matrix that should be eliminated, resulting in the formation of a new coefficient matrix.

(5) The aforementioned steps are repeated iteratively to ensure that the proportion of non-zero elements in each column of the coefficient matrix satisfies the criterion of being greater than the minimum probability factor. Based on the results of the coefficient matrix traversal mentioned above, the redundant parameters are adaptively eliminated. The optimal tomographic model is established as follows:

$$\begin{bmatrix} a_{k_1}^1 & a_{k_2}^1 & \cdots & a_{k_m}^1 \\ a_{k_1}^2 & a_{k_2}^2 & \cdots & a_{k_m}^2 \\ \vdots & \vdots & \ddots & \vdots \\ a_{k_1}^j & a_{k_2}^j & \cdots & a_{k_m}^j \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{k_m} \end{bmatrix} = \begin{bmatrix} SWV_1 \\ SWV_2 \\ \vdots \\ SWV_j \end{bmatrix} \quad (6)$$

where $a_{k_m}^j$ refers to the intercept of the j -th satellite signal passing through the k_m -th voxel. Note that k_m is smaller than n and j is smaller than l . Based on the positions between the

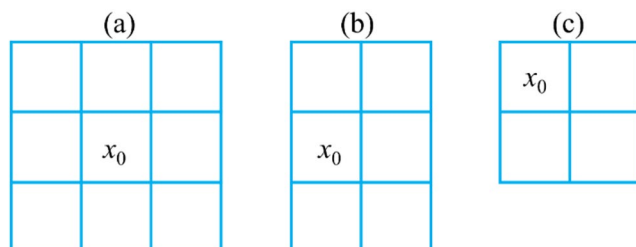


Fig. 1 Three examples of relationship between the voxels in the same horizontal layer

voxels corresponding to the estimated parameters in (6), the corresponding vertical weighted relationship is constructed as follows:

$$e^{\frac{(h_{k_m} - h_i)}{H_s}} \cdot x_{k_m} - x_i = 0 \quad (7)$$

where h_{k_m} and h_i refer to the height of the two voxels, and H_s is the water vapor scale height (Hao et al. 2025). H_s is derived from historical radiosonde or reanalysis data and represents the typical vertical decay scale of atmospheric water vapor, providing a physically meaningful constraint for the vertical relationship between voxels. Note that x_{k_m} and x_i are the parameters to be estimated in (6), and they are the closest voxels located in the same latitude and longitude region.

(6) The water vapor tomographic model, refined through the elimination of redundant parameters, is solved using the least squares method. Note that the algorithm described in the relevant literature is employed to determine the optimal weights for different satellite systems and different observations within the same systems (Guo et al. 2016; Zhao et al. 2018).

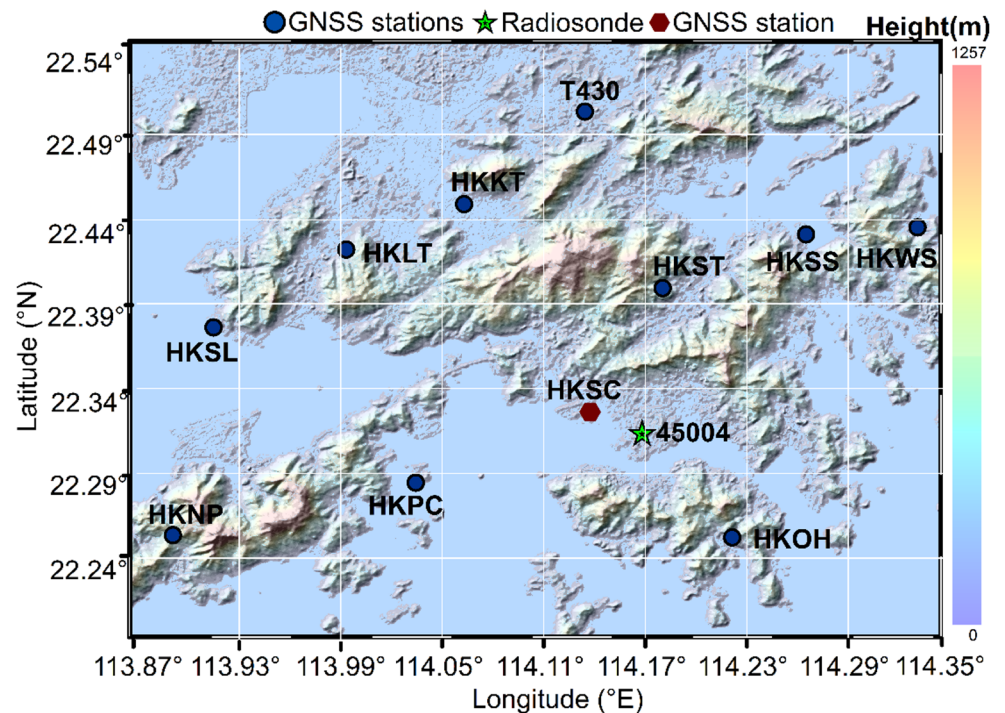
(7) The voxels appear three positional relationships in the same horizontal layer, as shown in Fig. 1, and their water vapor density can be expressed by a smoothing factor q which can be modelled according to Eq. (8). In practice, the smoothing factor q is determined by analyzing the spatial correlation of water vapor density in meteorological reanalysis data over the study region. This establishes the Laplace smoothing equation, which imposes a soft constraint assuming horizontal homogeneity of water vapor where direct observations are lacking.

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 - qx_0 = 0 \\ x_1 + x_2 + x_3 + x_4 + x_5 - qx_0 = 0 \\ x_1 + x_2 + x_3 - qx_0 = 0 \end{cases} \quad (8)$$

where x refers to the water vapor density of the corresponding voxel. All the voxels solved in Eq. (6) are traversed. If any of these solved voxels have neighboring voxels corresponding to redundant parameters, historical data from meteorological reanalysis is utilized to determine an appropriate smoothing factor for these voxels. Then the solved parameter obtained in Eq. (6) is substituted into the fitting formulas derived from historical data. This substitution generates a Laplace equation, which involves partial redundant parameters. These partial redundant parameters can be estimated by applying the least squares method.

(8) The redundant parameters solved in the previous step are considered as new solved parameters. The process described in step (7) is repeated iteratively until all the redundant parameters have been resolved.

Fig. 2 Distribution of GNSS stations, radiosonde site and the horizontal structure of the voxels in the study region



Experiment and analysis

Experiment description

To assess the performance of the proposed method, Hong Kong was selected as the research area to conduct the experiment. The area covers the longitude from 113.87°E to 114.35°E, the latitude from 22.19°N to 22.54°N and the altitude from 0 to 8 km, which contains the Hong Kong satellite Positioning Reference Station Network (SatRef). The research area is divided into 560 voxels, arranged in 8*7 voxel in the horizontal direction and consisting of 10 layers in the vertical direction. As shown in Figs. 2 and 3 GNSS sites represented by blue circles were utilized in the tomographic model to provide SWV values, and another GNSS site represented by a red pentagon and a radiosonde station represented by a green pentagram were adopted to assess the tomographic results.

The GAMIT 10.71 software was utilized to process the GNSS observation data, in which a double-differenced model was adopted to estimate the tropospheric parameters including ZTD and gradient parameters. Note that three International GNSS Service (IGS) stations (LHAZ, JFNG and URUM) were incorporated into the data processing to weaken the strong correlation of tropospheric parameters caused by the short baseline between GNSS receivers in the tomographic region. The sampling rate of the observations and the cut-off elevation angle were 30 s and 10°, respectively. The clock and orbit products from Center for Orbit Determination in Europe (CODE) were adopted. The

BASELINE and LC_AUTCLN modes were selected as the processing strategies, indicating that orbital parameters were fixed and the ionosphere-free linear combination of GNSS observations were processed, respectively. The GNSS data from a one-month period, specifically from Day of Year (DOY) 213 to 243 in 2022, were processed to conduct the water vapor tomography. Each tomographic solution covered a time period of 0.5 h. In this experiment, directly performing the least squares calculation on the tomographic equation constructed by (4) and corresponding constraints is

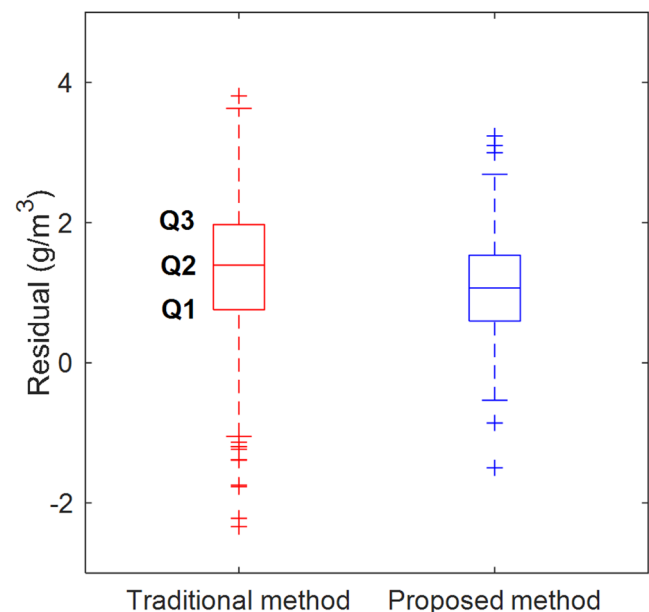


Fig. 3 Boxplots of the WVD residuals for the two methods

regarded as the traditional method. This approach represents the common foundational strategy used in many existing studies to handle the ill-posed problem, where the solution is sought for the full model despite its inherent deficiencies. Then the traditional method and the proposed method were utilized to conduct the water vapor tomography, and the estimated water vapor density obtained from each method were compared and analyzed.

Analysis of the ill condition

Figure 4(a) depicts a grayscale graph illustrating the voxels crossed by the SWV and the number of signal rays passing through each voxel in a tomographic solution at 12:00 UTC on DOY 213, in which the deepening of the grayscale represents an increase in the number of signal rays penetrating the voxel. In the lower layers closer to the ground, a significant number of voxels are not crossed by any signal rays, representing the first type of redundant parameters in water vapor tomography. In the upper layers, there are fewer voxels without SWV penetration, but many voxels have a lighter grayscale, indicating a lower number of signal rays passing through. To address this, the minimum probability factor from relevant literature was introduced to identify voxels with insufficient SWV penetration, which constitute the second type of redundant parameters in water vapor tomography. Figure 4(b) displays the distribution of these voxels, highlighting that a considerable number of voxels lack adequate signal ray penetration in both the lower and upper layers.

The situations mentioned-above in different layers were counted and plotted in Fig. 5 for the whole experiments, in which the upper panel denotes the number of voxels crossed by SWV in each layer, and the lower panel refers

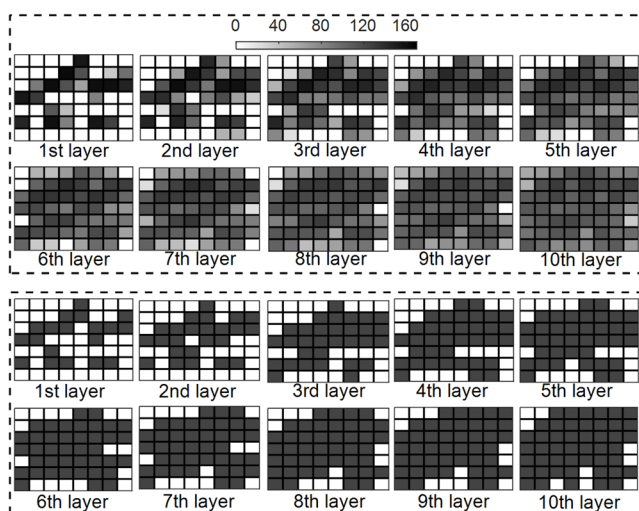


Fig. 4 Grayscale graph of number of signal rays penetrated each voxel and distribution of voxel with sufficient signal rays

to the number of voxels penetrated by sufficient SWV after applying the minimum probability. It is consistent with the results shown in Fig. 4, indicating that both types of redundant parameters decrease with the increase of layers. The average values of the redundant parameters are 38 and 10 for the first and 10th layer, respectively. In addition, the average value of the redundant parameters is 218 for the tomographic solutions during the experiment.

Validation of the slant water vapor

The SWV values at each site were calculated using the water vapor densities obtained from tomography and the distance of the signal rays passing through each voxel by (4), which is called as tomography-computed SWV. Then the GAMIT-estimated SWV was regarded as a reference to assess the SWV computed by the two methods. Figure 6 denotes the internal accuracy testing, meaning that the SWV values of the 10 GNSS sites used in the tomographic modelling were adopted. The blue and red dots refer to the proposed and traditional method, and it shows the change of tomography-computed vs. GAMIT-estimated SWV residuals with elevation angle. It is observed that the SWV differences increase as the elevation angle decreases for both methods. This is expected, as lower-elevation signals traverse longer and more complex paths through the atmosphere, accumulating more error and encountering greater heterogeneity in water vapor distribution. The root mean square error (RMSE) is 1.83 mm and the mean absolute error (MAE) is 0.92 mm for the traditional method, while RMSE and MAE values for the proposed method are 1.25 and 0.70 mm with approximately 31.7% and 23.9% improvements, respectively. The greater improvement at lower elevations can be seen in the figure, which suggests that the proposed method by eliminating unreliable parameters is effective in handling the complex signal paths where traditional methods struggle. For the proposed method, more than 93.5% of the SWV residual are located in the range of -2.0 to 2.0 mm, while the percentage was 88.2% for the traditional method. These percentages increase to 99.1% and 97.6% for the proposed and traditional methods when the range is expanded to -5.0 to 5.0 mm.

The tomography-computed SWV were mapped back to the zenith direction and their differences with the GAMIT-estimated SWV were calculated to normalize the SWV residuals. Figure 7 shows the statistical results of the residuals in the zenith direction at each GNSS site. It can be seen that the RMSE and MAE of the proposed method are better than those of the traditional method at each GNSS site. For the proposed method, the maximum and minimum RMSE/MAE are 0.94/0.67 and 0.33/0.19 mm, respectively, whereas these values are 1.30/0.84 and 0.59/0.34 for the

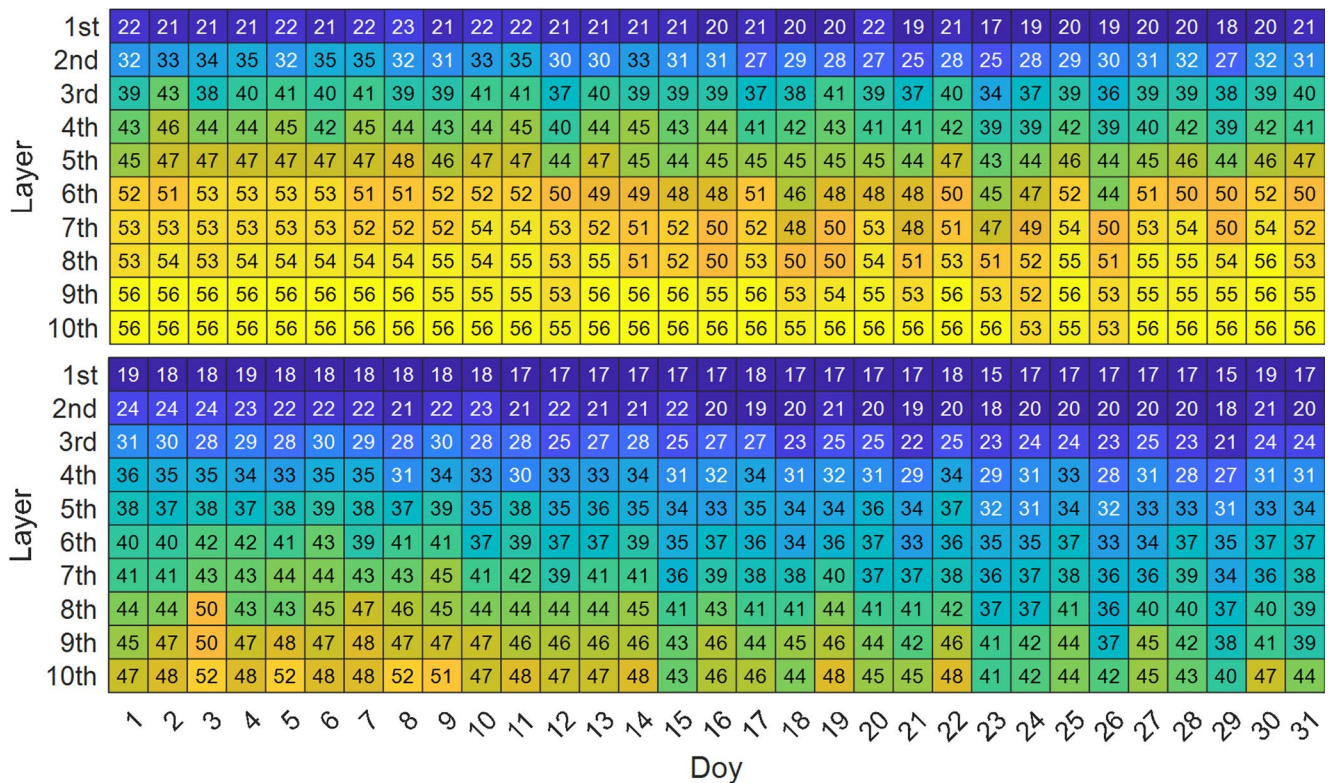


Fig. 5 The number of voxels penetrated by signal rays (upper) and sufficient signal rays in each layer during the experimental period

traditional method. The median of RMSE and MAE are displayed for 10 GNSS sites to highlight difference among the sites and the methods. Specifically, the median values are 0.63 and 0.41 for the proposed method which outperform the traditional method denoting values of 0.88 and 0.59 mm.

Furthermore, the external accuracy testing was performed using data from the HKSC site, which was not included in the tomographic modeling. Figure 8 shows the histogram for RMSE of SWV residuals (differences between the tomography-computed SWV and GAMIT-estimated SWV), in which the red and blue bars represent the traditional and proposed method, respectively. It is observed that all RMSE values are below 20 mm, with the value lower for the proposed method than for the traditional method at each day. The average values are 12.17 and 9.81 mm, respectively.

To further assess external accuracy, the SWV at HKSC site were grouped into individual elevation bins of 5, i.e., all SWVs with an elevation angle between 15 and 20° were evaluated as a single unit. Then the RMSE of each elevation bin was calculated, and the normalized RMSE was also computed to examine the dependence of relative errors in SWV on different elevations, which were shown in Fig. 9. It can be seen from the upper panel that the RMSE of SWV residuals decreases as the increasing elevation angle, which is consistent with the trend already shown in Fig. 6. The colors indicate a smaller RMSE and normalized RMSE can

be achieved by the proposed method than with the traditional method in each elevation bin. Note that the normalized RMSE remain almost constant over all elevation angle for the two methods. The proposed method delivers an average RMSE and normalized RMSE of 10.9 mm and 8.2%, performs better than the traditional method which exhibits average values of 13.2 mm and 10%, respectively.

Comparison with radiosonde data

The radiosonde data is well suited to assess the accuracy of the water vapor tomographic results, as it provides accurate water vapor density profiles. In this section, the water vapor density derived from the radiosonde at site 45,004 is utilized as reference to conduct the comparisons with the results of the two tomographic methods. Figure 10 shows the changes in water vapor density with altitudes for six selected tomographic solutions, obtained from derived from the traditional and proposed tomographic methods. In both methods, the water vapor density profiles exhibit a decreasing trend with increasing altitude. The profiles derived from the proposed method, depicted by the blue lines, demonstrate better consistency with the reference data obtained from the radiosonde. Specifically, the proposed method more accurately captures the steep decrease in water vapor density in the lower boundary layer and shows less oscillation in the

Fig. 6 Scatter diagram of the SWV residuals for the two methods in internal accuracy testing

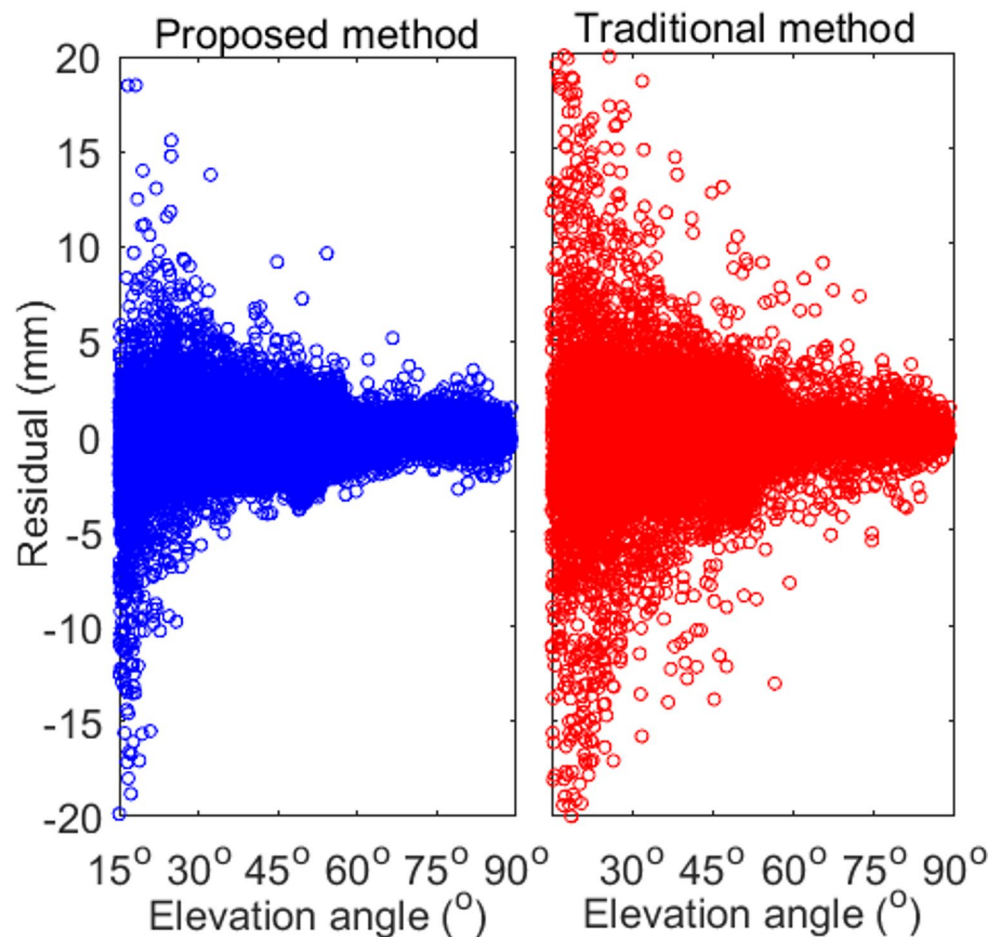
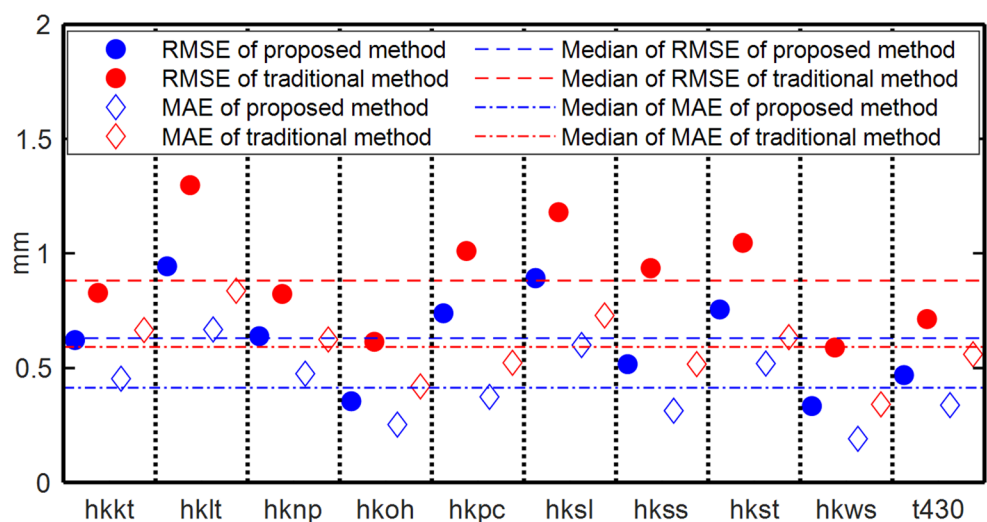


Fig. 7 Comparison of SWV residuals in zenith direction at each GNSS sites for the two methods



mid-troposphere, indicating a more physically realistic and stable reconstruction.

To further evaluate the tomographic results using the radiosonde data, the water vapor density residuals between the reference value and those obtained from the two tomographic methods were calculated for each day of the experimental period. Figure 3 presents the statistical

characteristics of these residuals using boxplots, where Q1 and Q3 represent the first and third quartiles, respectively, and Q2 denotes the second quartile located within the box. It can be seen that the proposed method exhibits a smaller range of bounds and box length (the range from Q1 to Q3) compared to the traditional method. This suggests that the proposed method yields a more favorable distributions of

Fig. 8 Histogram for RMSE of SWV residuals for the HKSC station during the experimental period

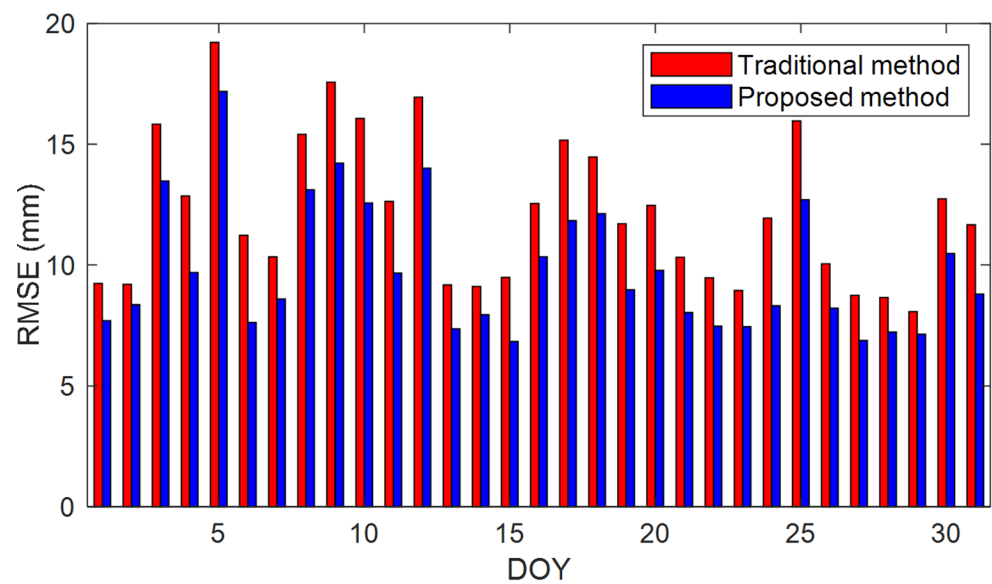


Fig. 9 Comparison of RMSE and normalized RMSE for the HKSC station in each elevation bin

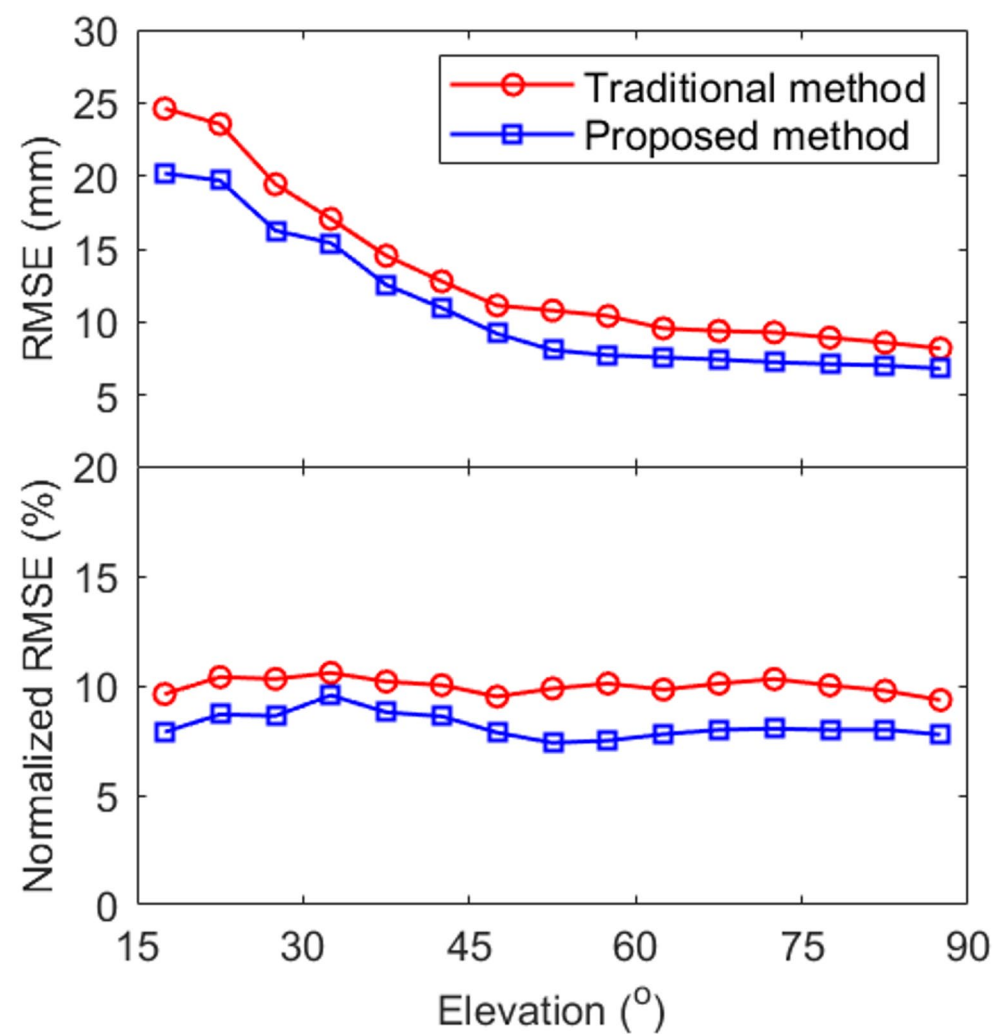


Fig. 10 Change in water vapor density with altitude for radiosonde data and the two methods in six selected tomographic solutions

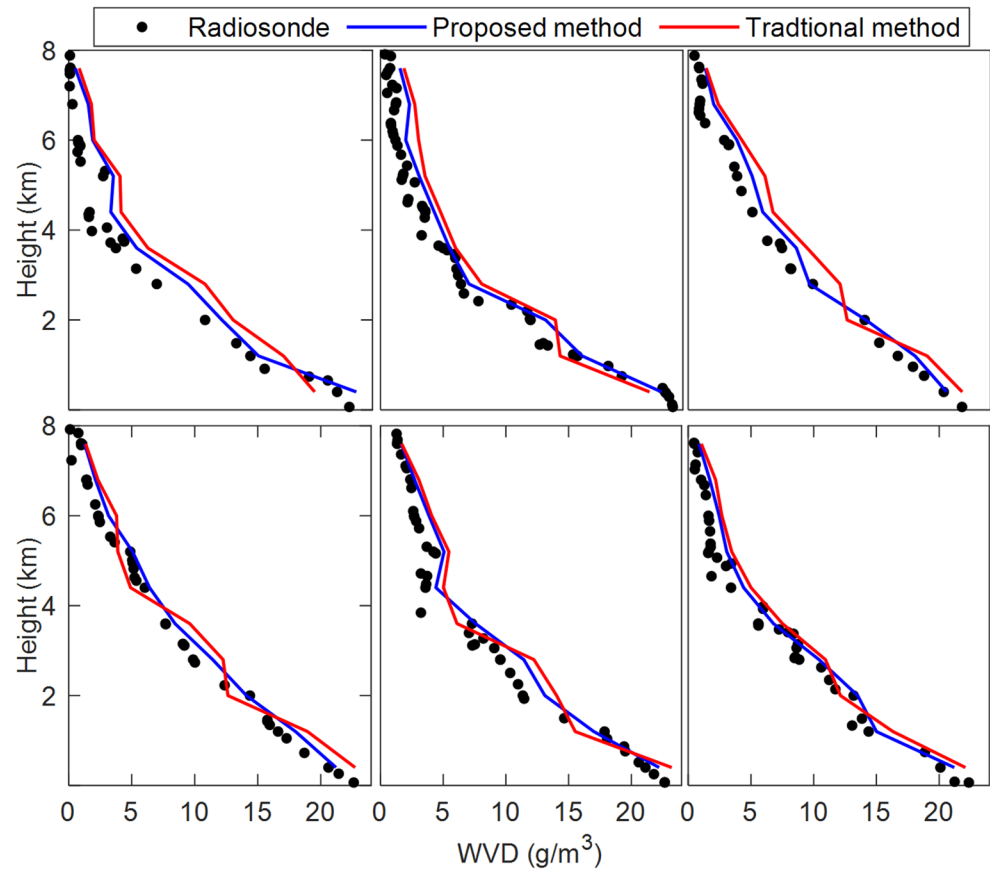


Table 1 RMSE and MAE values of the water vapor density from the two methods at different layers using radiosonde as references (values in g/m^3)

Layers	Proposed method		Traditional method	
	RMSE	MAE	RMSE	MAE
1	1.69	1.56	2.18	2.09
2	1.52	1.33	1.94	1.85
3	1.39	1.27	1.91	1.78
4	1.38	1.21	1.88	1.81
5	1.31	1.14	1.72	1.65
6	1.31	1.06	1.56	1.41
7	1.10	0.96	1.29	1.14
8	1.08	0.91	1.20	1.07
9	0.96	0.84	1.09	0.99
10	0.77	0.65	0.89	0.83
Average value	1.25	1.09	1.57	1.46

WVD residual than the traditional method. For the traditional method, the values of Q1, Q2 and Q3 are 0.76, 1.39 and 1.97 g/m^3 , while these values are 0.60, 1.07 and 1.54 g/m^3 for the proposed method. The upper and lower bounds show that the majority of residuals are concentrated in the range of -0.81 to 2.95 g/m^3 for the proposed method and -1.07 to 3.80 g/m^3 in the traditional method. The tighter distribution and lower median of the residuals from the

proposed method confirm its superior accuracy and reliability in estimating the 3D water vapor field.

Moreover, the difference in the WVD at different layers were computed for the two methods using the radiosonde data as references. The statistics including RMSE and MAE are listed in Table 1. It is observed that the proposed method outperforms the traditional method in each height layer, exhibiting lower RMSE and MAE values. For the proposed method, the mean RMSE and MAE values are 1.25 g/m^3 and 1.09 g/m^3 , respectively, while the corresponding values are 1.57 g/m^3 and 1.46 g/m^3 for the traditional method. The improvement is most pronounced in the lower layers (1–4), where water vapor concentration is highest and most variable. This demonstrates the method's enhanced capability to resolve the complex vertical structure of humidity, which is critical for meteorological applications.

Conclusion

An optimal method for water vapor tomography was proposed to estimate the three-dimensional water vapor density accurately, which is based on the parameter adaptation. In contrast to the traditional method which solves the complete tomographic equations containing redundant parameters,

the proposed method introduced the minimum probability factor of voxels being penetrated by signals, determined the number and position of the redundant parameters in the tomographic observation equation by iterating the tomographic coefficient matrix, and established the optimal tomographic model by eliminating the zero elements of the coefficient matrix and the redundant parameters. Then the water vapor density of all voxels was achieved by constructing a Laplace smoothing equation. This proposed method directly tackles the root cause of the ill-posed problem in water vapor tomography—discretization parameter redundancy—setting it apart from the traditional least-squares methods that rely on external data or constraints applied to an unoptimized model.

The GNSS data collected over HONG KONG from August 1, 2022 to August 31, 2022 were utilized to conduct the water vapor tomographic experiments to validate the proposed method. The redundant parameters in the traditional tomographic model were discussed and analyzed by the grayscale graph. In a comparison between the tomography-computed SWV and GAMIT-estimated SWV, the RMSE and MAE of the proposed method are 1.25 and 0.70 mm, outperforming the traditional method with an improvement of 31.7% and 23.9%, respectively. For the external accuracy testing using the HKSC site, the average RMSE and normalized RMSE are the indicators of concern, these values are 10.9 mm and 8.2%, and 13.2 mm and 10% for the proposed and traditional method, respectively. Moreover, the tomographic results achieved using the proposed method exhibited a better consistency with the water vapor profiles derived from the radiosonde data. Specifically, the proposed method achieved a mean RMSE and MEA values of 1.25 g/m³ and 1.09 g/m³, respectively, indicating improvements of approximately 20.4% and 25.3% compared to the traditional method.

The validation conducted in this study was performed under predominantly stable atmospheric conditions, and its performance remains to be assessed during intense meteorological events, such as typhoons or severe convective systems. Such evaluation is crucial to verify the method's capability in resolving the rapid, multi-scale dynamics of water vapor that drive high-impact weather, which is both the current limitation of this article and the direction for future research. In summary, the practical implications of this work are significant for the field of GNSS meteorology. The ability to retrieve more accurate and reliable water vapor distributions with high spatiotemporal resolution can greatly benefit Numerical Weather Prediction (NWP) by providing improved initial moisture fields for model assimilation. Furthermore, this detailed 3D information is crucial for understanding and nowcasting the initiation and evolution of convective weather systems, studying atmospheric

river events, and quantifying vertical moisture transport. The proposed method represents a valuable step towards operational GNSS meteorology applications.

Acknowledgements Thanks to the Hong Kong Geodetic Survey Services for providing the GNSS data of SatRef. We thank all anonymous reviewers for their valuable, constructive, and prompt comments.

Author contributions F.Y conceived and designed the study, F.Y, R.H and H. W conducted the experiment, F.Y, R.H, H.W, L. Z and L.W conducted the formal analysis, Z.L, H.W, J.Z, Y.W, L. W and R. C collected and pre-processed the data. F.Y, R.H and H.W wrote the main manuscript. All authors reviewed the manuscript.

Funding This study is supported by the National Natural Science Foundation of China (42204022), Fund of Major Special Projects of the “14th Five-Year Plan” Technological Breakthrough Program in Zhenhai District (2024006), the Open Research Fund of State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, Wuhan University (23P02), the Fundamental Research Funds for the Central Universities (2024ZKPYDC02, 2023ZKPYDC10), Open Fund of State Key Laboratory for Fine Exploration and Intelligent Development of Coal Resources(SKLCRSM24KFA13).

Data availability The GNSS and relative meteorological data can be download from the Hong Kong Satellite Positioning Reference Station Network (SatRef) (<https://www.geodetic.gov.hk/en/satref/satref.htm>). The radiosonde data were obtained from the Department of Atmospheric Science, University of Wyoming (<http://weather.uwyo.edu/upperair/sounding.html>). The other datasets generated and analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of Interests The authors declare no competing interests.

Ethics approval Not applicable.

Consent for publication Not applicable.

References

- Adavi Z, Mashhadi-Hossainali M (2014) 4D tomographic reconstruction of the tropospheric wet refractivity using the concept of virtual reference station, case study: Northwest of Iran. *Meteorol Atmos Phys* 126(3–4):193–205
- Bender M, Dick G, Ge M, Deng Z, Wickert J, Kahle H, Raabe A, Tetzlaff G (2011) Development of a GNSS water vapour tomography system using algebraic reconstruction techniques. *Adv Space Res* 47(10):1704–1720
- Benevides P, Nico G, Catalao J, Miranda P (2016) Bridging InSAR and GPS tomography: a new differential geometrical constraint. *IEEE Trans Geosci Remote Sens* 54(2):697–702
- Bevis M, Businger S, Herring T, Rocken C, Anthes R, Ware R (1992) GPS meteorology: remote sensing of atmospheric water vapor using the global positioning system. *J Geophys Res: Atmos* 97(D14):15787–15801
- Braun J, Rocken C, Meetrens C, Ware R (1999) Development of a water vapor tomography system using low cost L1 GPS receivers.

- 9th ARM science team meeting. US Department of Energy, San Antonio, pp 22–26
- Fan S, Chen Y, Peng X, Liu Z, Gu X (2021) Automatic vertical non-uniform stratification method for GNSS water vapor tomography. *J Geodesy Geodyn* 41(9):924–928
- Flores A, Ruffini G, Rius A (2000) 4D tropospheric tomography using GPS Slant wet delays. *Ann Geophys* 18(2):223–234
- Gradinarsky L, Jarlemark P (2004) Ground-Based GPS tomography of water vapor: analysis of simulated and real data. *J Meteorol Soc Jpn* 82(1B):551–560
- Guo J, Yang F, Shi J, Xu C (2016) An optimal method of global positioning system (GPS) troposphere tomography. *IEEE J Sel Top Appl Earth Obs Remote Sens* 12:5880–5887
- Haji-Aghajany S, Amerian Y, Verhagen S, Rohm W, Ma H (2020) An optimal troposphere tomography technique using the WRF model outputs and topography of the area. *Remote Sens* 12(9):1442
- Hao R, Yang F, Li Z, Zhang Y, Zhou L, Wang L (2025) ERA5-SH: A global grided scale height dataset for tropospheric parameters based on ERA5 reanalysis. *Sci Data* 12:381
- He L, Liu L, Su X, Xu C, Duan P (2015) Algebraic reconstruction algorithm of vapor tomography. *Acta Geodaetica Cartogr Sin* 44(1):32–38
- Heublein M, Alshawaf F, Erdnub B, Zhu X, Hinz S (2019) Compressive sensing reconstruction of 3D wet refractivity based on GNSS and InSAR observations. *J Geodesy* 93:197–217
- Huang L, Wang X, Xiong S, Li J, Liu L, Mo Z, Fu B, He H (2022) High-precision GNSS PWV retrieval using dense GNSS sites and in-situ meteorological observations for the evaluation of MERRA-2 and ERA5 reanalysis products over China. *Atmos Res* 276:106247
- Huang L, Liu W, Mo Z, Zhang H, Li J, Chen F, Liu L, Jiang W (2023) A new model for vertical adjustment of precipitable water vapor with consideration of the time-varying lapse rate. *GPS Solutions* 27(4):170
- Jiang P, Ye S, He S, Liu Y (2013) Ground-based GPS tomography of wet refractivity with adaptive Kalman filter. *Geomatics Inform Sci Wuhan Univ* 38(3):299–302
- Li J, Yang F, Yuan D, Wang H, Song S, Tan J, Wen Z (2024) Unraveling the accuracy enigma: investigating ZTD data precision in TUW-VMF3 and GFZ-VMF3 products using a comprehensive global GPS dataset. *IEEE Trans Geosci Remote Sens* 62:1–10
- Li J, Yang F, Fan L, Yuan D, Jing G, Wang Z, Xie J, Yang Y (2025) Improving the accuracy of zenith tropospheric delay in TUW-VMF3 and GFZ-VMF3 using feedforward neural networks. *GPS Solutions* 29:151
- Razin M, Voosoghi B (2020) Estimation of tropospheric wet refractivity using tomography method and artificial neural networks in Iranian case study. *GPS Solutions* 24(3):65
- Rohm W, Zhang K, Bosy J (2024) Limited constraint, robust Kalman filtering for GNSS troposphere tomography. *Atmospheric Meas Technique* 7(5):1475–1486
- Sa A, Rohm W, Fernandes R, Trzcina E, Bos M, Bento F (2021) Approach to leveraging real-time GNSS tomography usage. *J Geodesy* 95(1):1–21
- Saastamoinen J (1972) Atmospheric correction for the troposphere and stratosphere in radio ranging satellites. *Use Artif Satell Geod* 15:247–251
- Shafei M, Hossainali M (2020) Application of the GPS reflected signals in tomographic reconstruction of the wet refractivity in Italy. *J Atmos Solar Terr Phys* 207:105348
- Song S, Zhu W, Ding J (2006) 3D water vapor tomography with Shanghai GPS network to improve forecasted moisture field. *Chin Sci Bull* 51:607–614
- Sun Y, Yang F, Liu M, Li Z, Gong X, Wang Y (2023) Evaluation of the weighted mean temperature over China using multiple reanalysis data and radiosonde. *Atmos Res* 285:106664
- Trzcina E, Rohm W (2019) Estimation of 3D wet refractivity by tomography, combining GNSS and NWP data: first results from assimilation of wet refractivity into NWP. *Q J R Meteorol Soc* 145:1034–1051
- Wan R, Zheng G, Yu S, Fu Z (2013) A study of the ground-based GPS 3D water vapor tomography with radiosonde vertical constraining. *71(2):* 318–331
- Wang H, Yang F, Zheng J, Wang Z, Chen W, Xie J (2025) From GNSS zenith tropospheric delay to precipitable water vapor: accuracy assessment using in-situ and reanalysis meteorological data over China. *IEEE J Sel Top Appl Earth Observations Remote Sens* 18:14582–14593
- Xia P, Ye S, Jiang P, Pan L, Guo M (2018) Assessing water vapor tomography in Hong Kong with improved vertical and horizontal constraints. *Ann Geophys* 36(4):969–978
- Yang F, Guo J, Shi J, Zhou L, Xu Y, Chen M (2018) A method to improve the distribution of observations in GNSS water vapor tomography. *Sensors* 18(8):2526
- Yang F, Guo J, Shi J, Meng X, Zhao Y, Zhou L (2020a) A GPS water vapor tomography method based on a genetic algorithm. *Atmos Meas Tech* 13:355–371
- Yang F, Meng X, Guo J, Yuan D, Chen M (2021) Development and evaluation of the refined zenith tropospheric delay (ZTD) models. *Satell Navig* 2:21
- Yang F, Wang L, Li Z, Tang W, Meng X (2022) A weighted mean temperature (T_m) augmentation method based on global latitude zone. *GPS Solutions* 26:141
- Yang F, Sun Y, Meng X, Guo J, Gong X (2023a) Assessment of tomographic window and sampling rate effects on GNSS water vapor tomography. *Satell Navig* 4:7
- Yang F, Gong X, Wang Y, Liu M, Li J, Xu T, Hao R (2023b) GNSS water vapor tomography based on Kalman filter with optimized noise covariance. *GPS Solute* 27:181
- Yao Y, Zhao Q (2017b) A novel, optimized approach of voxel division for water vapor tomography. *Meteorol Atmos Phys* 129(1):57–70
- Yao Y, Zhang S, Kong J (2017a) Research progress and prospect of GNSS space environment science. *Acta Geodaetica Cartogr Sin* 46(10):1408–1420
- Ye S, Xia P, Cai C (2016) Optimization of GPS water vapor tomography technique with radiosonde and cosmic historical data. *Ann Geophys* 34(9):789–799
- Zhang B, Fan Q, Yao Y, Xu C, Li X (2017) An improved tomography approach based on adaptive smoothing and ground meteorological observations. *Remote Sens* 9:886
- Zhang W, Zhang S, Ding N, Zhao Q (2020a) A tropospheric tomography method with a novel height factor model including two parts: isotropic and anisotropic height factors. *Remote Sens* 12(11):1848
- Zhang W, Zhang S, Zuo D, Ding N, Liu X (2020b) Adaptive algebraic reconstruction algorithms for GNSS water vapor tomography. *Geomatics Inform Sci Wuhan Univ* 46(9):1318–1327
- Zhang W, Zhang S, Chang G, Ding N, Wang X (2021a) A new hybrid observation GNSS tomography method combining the real and virtual inverted signals. *J Geodesy* 95:128
- Zhang W, Zhang S, Zheng N (2021b) A new integrated method of GNSS and MODIS measurements for tropospheric water vapor tomography. *GPS Solutions* 25(2):79
- Zhao Q, Yao Y (2017) An improved troposphere tomographic approach considering the signals coming from the side face of the tomographic area. 35:87–95
- Zhao Q, Yao Y, Yao W (2018) Troposphere water vapour tomography: a horizontal parameterized approach. *Remote Sens* 10:1241
- Zhao Q, Yao Y, Yao W (2021) A method to Establish the tomography model considering the data of GNSS stations outside the research area. *Acta Geodaetica Cartogr Sin* 50(3):285–294

Zhao Q, Du Z, Li Z, Yao W, Yao Y (2022) Two-step precipitable water vapor fusion method. *IEEE Trans Geosci Remote Sens* 60:5801510

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.

Terms and Conditions

Springer Nature journal content, brought to you courtesy of Springer Nature Customer Service Center GmbH (“Springer Nature”).

Springer Nature supports a reasonable amount of sharing of research papers by authors, subscribers and authorised users (“Users”), for small-scale personal, non-commercial use provided that all copyright, trade and service marks and other proprietary notices are maintained. By accessing, sharing, receiving or otherwise using the Springer Nature journal content you agree to these terms of use (“Terms”). For these purposes, Springer Nature considers academic use (by researchers and students) to be non-commercial.

These Terms are supplementary and will apply in addition to any applicable website terms and conditions, a relevant site licence or a personal subscription. These Terms will prevail over any conflict or ambiguity with regards to the relevant terms, a site licence or a personal subscription (to the extent of the conflict or ambiguity only). For Creative Commons-licensed articles, the terms of the Creative Commons license used will apply.

We collect and use personal data to provide access to the Springer Nature journal content. We may also use these personal data internally within ResearchGate and Springer Nature and as agreed share it, in an anonymised way, for purposes of tracking, analysis and reporting. We will not otherwise disclose your personal data outside the ResearchGate or the Springer Nature group of companies unless we have your permission as detailed in the Privacy Policy.

While Users may use the Springer Nature journal content for small scale, personal non-commercial use, it is important to note that Users may not:

1. use such content for the purpose of providing other users with access on a regular or large scale basis or as a means to circumvent access control;
2. use such content where to do so would be considered a criminal or statutory offence in any jurisdiction, or gives rise to civil liability, or is otherwise unlawful;
3. falsely or misleadingly imply or suggest endorsement, approval, sponsorship, or association unless explicitly agreed to by Springer Nature in writing;
4. use bots or other automated methods to access the content or redirect messages
5. override any security feature or exclusionary protocol; or
6. share the content in order to create substitute for Springer Nature products or services or a systematic database of Springer Nature journal content.

In line with the restriction against commercial use, Springer Nature does not permit the creation of a product or service that creates revenue, royalties, rent or income from our content or its inclusion as part of a paid for service or for other commercial gain. Springer Nature journal content cannot be used for inter-library loans and librarians may not upload Springer Nature journal content on a large scale into their, or any other, institutional repository.

These terms of use are reviewed regularly and may be amended at any time. Springer Nature is not obligated to publish any information or content on this website and may remove it or features or functionality at our sole discretion, at any time with or without notice. Springer Nature may revoke this licence to you at any time and remove access to any copies of the Springer Nature journal content which have been saved.

To the fullest extent permitted by law, Springer Nature makes no warranties, representations or guarantees to Users, either express or implied with respect to the Springer nature journal content and all parties disclaim and waive any implied warranties or warranties imposed by law, including merchantability or fitness for any particular purpose.

Please note that these rights do not automatically extend to content, data or other material published by Springer Nature that may be licensed from third parties.

If you would like to use or distribute our Springer Nature journal content to a wider audience or on a regular basis or in any other manner not expressly permitted by these Terms, please contact Springer Nature at

onlineservice@springernature.com